

REMARKS ON WEYL INVARIANT P-BRANES AND D_p-BRANES

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Abstract

A mechanism to find different Weyl invariant p-branes and D_p-branes actions is explained. Our procedure clarifies the Weyl invariance for such systems. Besides, by considering gravity-dilaton effective action in higher dimensions we also derive a Weyl invariant action for p-branes. We argue that this derivation provides a geometrical scenario for the Weyl invariance of p-branes. Our considerations can be extended to the case of super-p-branes.

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1.- INTRODUCTION

It is known that, in string theory, the Weyl invariance of the Polyakov action plays a central role [1]. In fact, in string theory subjects such as moduli space, Teichmuler space and critical dimensions, among many others, are consequences of the local diff x Weyl symmetry in the partition function associated to the Polyakov action.

In the early eighties there was a general believe that the Weyl invariance was the key symmetry to distinguish string theory from other p-branes. However, in 1986 it was noticed that such an invariance may be also implemented to any p-brane and in particular to the 3-brane [2]. The formal relation between the Weyl invariance and p-branes was established two years later independently by a number of authors [3]. It spites of its possible relevance in M-theory, it seems that except for some works [4] the subject has been very much ignored. Recently, however, the importance of the Weyl invariance has been revived in connection with 2d-gravity [5] and Dp-branes [6].

On the other hand, in a recent work [7] it was proved that it is possible to obtain the p-brane action from a gravity-dilaton effective action in higher dimensions. This work has some relevance, among other things, because it opens the possibility to understand symmetries of the p-branes from a geometrical point of view. In particular, it may be of special interest to understand the origin of the Weyl invariance for p-branes from a geometrical scenario. In this article, we prove that using a particular ansatz for the metric, a gravity-dilaton action in $D + 1$ dimensions is reduced to a Weyl invariant p-brane action, clarifying with this the geometric origin of Weyl invariance for p-branes. As reference [7], the present work may be of special interest in the Randall-Sundrum scenario [8-9], string theory [10] and M-theory [11].

We start making some general remarks on the Weyl invariant p-branes and D-branes. Specifically, we show a mechanism to obtain the Weyl invariant actions for p-branes and D-branes from the Dirac-Nambu-Goto p-brane action and the Born-Infeld Dp-brane action, respectively. We also apply a similar mechanism to the case of null p-branes and derive a

Weyl invariant action for such system.

The plan of this work is as follows. In section 2, we show a mechanism to obtain Weyl invariant p-branes from the Dirac-Nambu-Goto p-brane action. In section 3, we apply a similar procedure in order to obtain Weyl invariant Dp-branes from the Born-Infeld Dp-brane action. In section 4, we derive a Weyl invariant action for null p-branes. In section 5, we obtain a Weyl invariant p-brane action from a gravity-dilaton effective action. Finally, in section 6 we make some final comments.

2.- WEYL INVARIANT P-BRANES FROM DIRAC-NAMBU-GOTO P-BRANE ACTION

Consider a p-brane moving in a $d + 1$ -dimensional Minkowski space-time. The evolution of such a system may be described by using the $d + 1$ -scalar field coordinates $x^{\hat{\mu}}(\xi^A)$ where $\hat{\mu} = 0, 1, \dots, d$ and ξ^A , with $A = 0, 1, \dots, p$, are arbitrary parameters.

The Dirac-Nambu-Goto type action for p-branes is

$$S_p^{(1)} = -T_p \int d^{p+1}\xi \sqrt{-h}, \quad (1)$$

where $h = \det(h_{AB})$, with

$$h_{AB} = \partial_A x^{\hat{\mu}} \partial_B x^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}}, \quad (2)$$

and T_p is a fundamental constant measuring the inertia of the p-brane. Here,

$$\eta_{\hat{\mu}\hat{\nu}} = \text{diag}(-1, 1, \dots, 1) \quad (3)$$

is the Minkowski metric.

Let us now consider a symmetric auxiliary world-volume metric $g_{AB} = g_{AB}(\xi^A)$ and let us assume that the integrand $\sqrt{-h}$ in (1) is replaced by $\sqrt{-g}$ where $g = \det(g_{AB})$. The question is how to get an equivalent action to (1) with $\sqrt{-g}$ as the integrand instead of $\sqrt{-h}$. Clearly, the integrands $\sqrt{-g}$ and $\sqrt{-h}$ are equivalent if we assume the constraint

$$h_{AB} - g_{AB} = 0. \quad (4)$$

So, by introducing the lagrange multipliers $\Lambda^{AB} = \Lambda^{BA}$ we discover the action

$$S_p^{(2)} = -T_p \int d^{p+1} \xi (\sqrt{-g} + \Lambda^{AB} (h_{AB} - g_{AB})) \quad (5)$$

which is equivalent to $S_p^{(1)}$. In fact, the Λ^{AB} field equation implies the constraint (4) which allows to reduce (5) to (1). The point is that g_{AB} and Λ^{AB} provide to the action $S_p^{(2)}$ with more degrees of freedom than $S_p^{(1)}$, but these additional degrees of freedom can be eliminated by means of (4) reducing $S_p^{(2)}$ to $S_p^{(1)}$.

We may obtain another equivalent action to $S_p^{(1)}$ if instead of varying Λ^{AB} in (5) we make variations of g_{AB} . We find that g_{AB} field equation is

$$\frac{1}{2} \sqrt{-g} g^{AB} - \Lambda^{AB} = 0. \quad (6)$$

This equation allows to eliminate Λ^{AB} from (5). Thus, we obtain the alternative action

$$S_p^{(3)} = -\frac{T_p}{2} \int d^{p+1} \xi \sqrt{-g} (g^{AB} h_{AB} - (p-1)) \quad (7)$$

which is the familiar Polyakov type action for p-branes. Thus, starting from the Dirac-Nambu-Goto action (1) we have found a mechanism to derive the Polyakov action for p-branes (7). The procedure to derive $S_p^{(1)}$ starting from $S_p^{(3)}$ is well known. One first derive from $S_p^{(3)}$ the g^{AB} field equation

$$h_{AB} - \frac{1}{2} g_{AB} (g^{CD} h_{CD} - (p-1)) = 0. \quad (8)$$

Multiplying this expression by g^{AB} , it is not difficult to obtain, for $p \neq 1$, the formula

$$g^{AB} h_{AB} = p + 1. \quad (9)$$

Substituting (9) into (8) leads us to the constraint (4) which allows to reduce (7) to (1).

Let us now follow a slightly different procedure. Consider the rescaling of the metric $g_{AB} \rightarrow \Phi g_{AB}$, where $\Phi = \Phi(\xi)$ is a scale factor. If we apply this transformation to (5), leaving Λ^{AB} and h_{AB} unchanged, we obtain

$$S_p^{(\hat{2})} = -T_p \int d^{p+1}\xi (\Phi^{\frac{p+1}{2}} \sqrt{-g} + \Lambda^{AB} (h_{AB} - \Phi g_{AB})). \quad (10)$$

Varying g_{AB} in (10) leads to

$$\frac{1}{2} \sqrt{-g} \Phi^{\frac{p+1}{2}} g^{AB} - \Phi \Lambda^{AB} = 0. \quad (11)$$

Solving for Λ^{AB} gives

$$\Lambda^{AB} = \frac{1}{2} \sqrt{-g} \Phi^{\frac{p-1}{2}} g^{AB}. \quad (12)$$

Substituting this result back into (10) yields

$$S_p^{W_1} = -\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-g} (\Phi^{\frac{p-1}{2}} g^{AB} h_{AB} - (p-1) \Phi^{\frac{p+1}{2}}). \quad (13)$$

Note that if we set $\Phi = 1$ then (13) is reduced to (7). Therefore, the action $S_p^{W_1}$ is a generalization of $S_p^{(3)}$. We also discover that (13) is invariant under the transformations

$$\Phi \rightarrow \lambda^{-1} \Phi \quad (14)$$

and

$$g_{AB} \rightarrow \lambda g_{AB}. \quad (15)$$

where $\lambda(\xi)$ is an arbitrary function. Of course, (15) is a Weyl transformation. In order to clarify this point let us integrate out Φ from (13). We find the formula

$$\Phi = \frac{g^{AB} h_{AB}}{p+1}. \quad (16)$$

Substituting this result into (13) leads to

$$S_p^{(4)} = -\frac{T_p}{(p+1)^{\frac{p+1}{2}}} \int d^{p+1}\xi \sqrt{-g} (g^{AB} h_{AB})^{\frac{p+1}{2}}. \quad (17)$$

We recognize this action as the Weyl invariant action for p-branes [2]-[3]. In fact, the action $S_p^{(4)}$ is invariant under the Weyl transformation (15). Moreover, it is straightforward to see that the Weyl invariant action $S_p^{(1)}$ is recovered upon eliminating g_{AB} from $S_p^{(4)}$ via its field equation, showing with this that $S_p^{(4)}$ is classically equivalent to $S_p^{(1)}$.

If we redefine Φ as

$$\Phi = \phi^{\frac{2}{p+1}} \quad (18)$$

we find that (13) becomes

$$S_p^{W_2} = -\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-g} (\phi^{\frac{p-1}{p+1}} g^{AB} h_{AB} - (p-1)\phi) \quad (19)$$

which is the action proposed in reference [6].

An interesting possibility arises if instead of using (18) we redefine Φ as

$$\Phi = \varphi^{\frac{2}{p-1}}, \quad (20)$$

with $p-1 \neq 0$. In fact, in this case the action (13) becomes

$$S_p^{W_3} = -\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-g} (\varphi g^{AB} h_{AB} - (p-1)\varphi^{\frac{p+1}{p-1}}). \quad (21)$$

This action is interesting because in the second term of (21) the scalar field φ gives a power potential. The case $p=1$ is solved if in (21) we set $\varphi=1$.

3.- WEYL INVARIANT Dp-BRANES FROM BORN-INFELD Dp-BRANE ACTION

In this section, we shall show that a similar procedure, as in the previous section, can be applied to the case of Dp-branes. Consider the Born-Infeld action for Dp-branes

$$S_{Dp}^{(1)} = -T_p \int d^{p+1}\xi \sqrt{-f}, \quad (22)$$

where T_p is the Dp-brane “tension” and $f = \det(f_{AB})$ with

$$f_{AB} = h_{AB} - \mathcal{F}_{AB}.$$

Here, \mathcal{F}_{AB} is a two form given by

$$\mathcal{F}_{AB} = F_{AB} - B_{AB}, \quad (23)$$

where F_{AB} is the field strength

$$F_{AB} = \partial_A A_B - \partial_B A_A \quad (24)$$

of a $U(1)$ gauge field A_B and B_{AB} is the NS antisymmetric two form field. Note that if $\mathcal{F}_{AB} = 0$ then (22) is reduced to (1) and therefore (22) is a generalization of the Dirac-Nambu-Goto type action for p-branes.

Following a similar procedure as in the previous section we introduce an auxiliary metric world-volume metric $s_{AB} = s_{AB}(\xi^A)$. In contrast to g_{AB} the second rank tensor s_{AB} has no symmetries. If we consider the constraint

$$f_{AB} - s_{AB} = 0 \quad (25)$$

and introduce the lagrange multipliers Σ^{AB} , it is straightforward to show that the action

$$S_{Dp}^{(2)} = -T_p \int d^{p+1} \xi (\sqrt{-s} + \Sigma^{AB} (f_{AB} - s_{AB})) \quad (26)$$

is equivalent to $S_{Dp}^{(1)}$.

Varying s_{AB} in $S_{Dp}^{(2)}$ we find the formula

$$\frac{1}{2} \sqrt{-s} s^{AB} - \Sigma^{AB} = 0. \quad (27)$$

This equation allows to eliminate Σ^{AB} from (26). We obtain

$$S_{Dp}^{(3)} = -\frac{T_p}{2} \int d^{p+1} \xi \sqrt{-s} (s^{AB} f_{AB} - (p-1)) \quad (28)$$

which is the analogue of the Polyakov type action for p-branes (see Ref. [12]). Using the field equation for s_{AB} one can recover the action $S_{Dp}^{(1)}$ from $S_{Dp}^{(3)}$. So, $S_{Dp}^{(1)}$ and $S_{Dp}^{(3)}$ are classically equivalent actions.

Now, consider the rescaling $s_{AB} \rightarrow \Phi s_{AB}$, where $\Phi(\xi)$ is again a scale factor function and apply this transformation to (26), leaving Σ^{AB} and f_{AB} unchanged. We obtain

$$S_{Dp}^{(\hat{2})} = -T_p \int d^{p+1} \xi (\Phi^{\frac{p+1}{2}} \sqrt{-s} + \Sigma^{AB} (f_{AB} - \Phi s_{AB})). \quad (29)$$

Varying s_{AB} we find

$$\frac{1}{2}\sqrt{-g}\Phi^{\frac{p+1}{2}}s^{AB}-\Phi\Sigma^{AB}=0. \quad (30)$$

Solving for Λ^{AB} and substituting the result back into (29) yields

$$S_{Dp}^{W_1} = -\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-s}(\Phi^{\frac{p-1}{2}}s^{AB}f_{AB} - (p-1)\Phi^{\frac{p+1}{2}}). \quad (31)$$

We discover that $S_{Dp}^{W_1}$ is invariant under the transformations

$$\Phi \rightarrow \lambda^{-1}\Phi \quad (32)$$

and

$$s_{AB} \rightarrow \lambda s_{AB}. \quad (33)$$

Integrating Φ in (31) we find the field equation

$$\Phi = \frac{s^{AB}f_{AB}}{p+1}. \quad (34)$$

Substituting this result into (31) leads to

$$S_{Dp}^{(4)} = -\frac{T_p}{(p+1)^{\frac{p+1}{2}}} \int d^{p+1}\xi \sqrt{-s}(s^{AB}f_{AB})^{\frac{p+1}{2}} \quad (35)$$

which is a generalization of the Weyl invariant action for p-branes (17) (see Ref. [13]).

If we redefine Φ as

$$\Phi = \phi^{\frac{2}{p+1}}, \quad (36)$$

we find that (31) becomes

$$S_{Dp}^{W_2} = -\frac{T_p}{2} \int d^{p+1}\xi \sqrt{-s}(\phi^{\frac{p-1}{p+1}}s^{AB}f_{AB} - (p-1)\phi). \quad (37)$$

Similar, if we redefine Φ as

$$\Phi = \varphi^{\frac{2}{p-1}}, \quad (38)$$

with $p-1 \neq 0$, we find

$$S_{Dp}^{W_3} = -\frac{T_p}{2} \int d^{p+1} \xi \sqrt{-s} (\varphi s^{AB} f_{AB} - (p-1) \varphi^{\frac{p+1}{p-1}}). \quad (39)$$

It seems that the actions $S_{Dp}^{W_1}$, $S_{Dp}^{W_2}$ or $S_{Dp}^{W_3}$ have not been considered in the literature. In reference [6] a Weyl invariant action for Dp-branes was proposed, but the authors used the action of Zeid and Hull [14] rather than $S_{Dp}^{(3)}$ action.

4.- WEYL INVARIANT NULL P-BRANES

The procedure may also work out for null p-branes. Let us show in this section this fact. Null p-branes are defined for the case $T_p = 0$. So, the action (1) is not appropriate for this case. The key is first to write h as

$$h = \sigma^{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} \sigma_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}}, \quad (40)$$

where

$$\sigma^{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} = \frac{1}{[(p+1)!]^{\frac{1}{2}}} \varepsilon^{A_1 \dots A_{p+1}} \partial_{A_1} x^{\hat{\mu}_1} \dots \partial_{A_{p+1}} x^{\hat{\mu}_{p+1}}. \quad (41)$$

Then, one can show that the action (1) is equivalent to

$$S_p^{(1)} = \int d^{p+1} \xi (\sigma^{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} p_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} - \frac{\gamma}{2} (p^{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} p_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} + T_p^2)). \quad (42)$$

where γ is a lagrange multiplier. If we eliminate $p_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}}$ from this action we get

$$S_p^{(1)} = \frac{1}{2} \int d^{p+1} \xi (\gamma^{-1} \sigma^{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} \sigma_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}} - \gamma T_p^2). \quad (43)$$

By eliminating γ from (43) leads us to recover action (1). The importance of (42) or (43) is that it now makes sense to set $T_p = 0$. In this case (43) is reduced to the Schild type null p-brane action [15]

$$S_{Np}^{(1)} = \frac{1}{2} \int d^{p+1} \xi \gamma^{-1} h. \quad (44)$$

Now, consider the equivalent action

$$S_{Np}^{(2)} = \frac{1}{2} \int d^{p+1} \xi (\gamma^{-1} g + \Lambda^{AB} (h_{AB} - g_{AB})). \quad (45)$$

The g_{AB} field equation is

$$\gamma^{-1} g g^{AB} - \Lambda^{AB} = 0. \quad (46)$$

Using this expression we learn that (45) is reduced to

$$S_{Np}^{(2)} = \frac{1}{2} \int d^{p+1} \xi \gamma^{-1} g (g^{AB} h_{AB} - p). \quad (47)$$

Considering the rescaling $g_{AB} \rightarrow \Phi g_{AB}$ we see that the action (45) becomes

$$S_{Np}^{(\hat{2})} = \int d^{p+1} \xi (\Phi^{p+1} \gamma^{-1} g + \Lambda^{AB} (h_{AB} - \Phi g_{AB})). \quad (48)$$

Now, we have

$$\gamma^{-1} \Phi^{p+1} g g^{AB} - \Phi \Lambda^{AB} = 0. \quad (49)$$

Therefore, we obtain

$$S_{Np}^{W_1} = \int d^{p+1} \xi (\gamma^{-1} g (\Phi^p g^{AB} h_{AB} - p \Phi^{p+1})). \quad (50)$$

Two alternative actions are

$$S_{Np}^{W_2} = \int d^{p+1} \xi (\gamma^{-1} g (\phi^{\frac{p}{p+1}} g^{AB} h_{AB} - p \phi)) \quad (51)$$

and

$$S_p^{W_3} = \int d^{p+1} \xi (\gamma^{-1} g (\varphi g^{AB} h_{AB} - p \varphi^{\frac{p+1}{p}})), \quad (52)$$

depending if we redefine Φ as $\Phi = \phi^{\frac{1}{p+1}}$ or $\Phi = \varphi^{\frac{1}{p}}$, respectively.

In (52) varying φ we find the field equation

$$\varphi = \frac{1}{(p+1)^p} (g^{AB} h_{AB})^p. \quad (53)$$

Therefore, $S_{Np}^{W_3}$ becomes

$$S_{Np}^{(4)} = \frac{(1-p)}{(1+p)^p} \int d^{p+1} \xi \gamma^{-1} g(g^{AB} h_{AB})^{p+1} \quad (54)$$

which is the Weyl invariant action for null p-branes [16].

5.- WEYL INVARIANT P-BRANES FROM A GRAVITY-DILATON ACTION

Here, we shall closely follow reference [7]. Our starting point is the gravity-dilaton effective action with cosmological constant:

$$S = -\frac{1}{16\pi G_{D+1}} \int d^{D+1} y \sqrt{-g} e^{-\phi} (\varphi(R + (\nabla\phi)^2) + 2\Lambda\varphi^a), \quad (55)$$

where G_{D+1} is the Newton constant in $D+1$ dimensions, $\phi = \phi(y^\alpha)$ is the dilaton field, Λ and a are constants, φ is a lagrange multiplier and R is the Ricci scalar obtained from the Riemann tensor

$$R_{\nu\alpha\beta}^\mu = \Gamma_{\nu\beta,\alpha}^\mu - \Gamma_{\nu\alpha,\beta}^\mu + \Gamma_{\sigma\alpha}^\mu \Gamma_{\nu\beta}^\sigma - \Gamma_{\sigma\beta}^\mu \Gamma_{\nu\alpha}^\sigma \quad (56)$$

and the metric tensor $g_{\alpha\beta}$, with $\alpha, \beta = 0, 1, \dots, D$. Here, $\Gamma_{\alpha\beta}^\mu$ is the Christoffel symbol:

$$\Gamma_{\alpha\beta}^\mu = \frac{1}{2} g^{\mu\nu} (g_{\nu\alpha,\beta} + g_{\nu\beta,\alpha} - g_{\alpha\beta,\nu}). \quad (57)$$

Consider the ansatz

$$g_{AB} = \tilde{g}_{AB}(y^C),$$

$$g_{ij} = a_k(y^C) a_l(y^C) \eta_{ij}^{kl}, \quad (58)$$

$$g_{Ai} = 0.$$

Here, the indices $A, B, \dots etc.$ run from 0 to p , the indices $i, j, \dots etc.$ run from $p+1$ to D and the only non-vanishing terms of η_{ij}^{kl} are

$$\eta_{ij}^{kl} = 1, \text{ when } k = l = i = j. \quad (59)$$

Assume that

$$\phi = \phi(y^C) \quad (60)$$

and

$$\varphi(y^C). \quad (61)$$

Using (56), (57) and (58) one can compute the non-vanishing Christoffel symbols and the Reimann tensor. The result for the Ricci scalar $R = g^{\mu\nu} R_{\mu\nu}$ (see Ref. [7] for detail computation, and Ref. [17] for the case of $p = 0$) is

$$\begin{aligned} R = & -2a^i D^A \partial_A a_i - a^i a^j \partial_A a_i \partial^A a_j \\ & + a^i a^j \partial_A a_k \partial^A a_l \eta_{ij}^{kl} + \tilde{R}, \end{aligned} \quad (62)$$

where \tilde{R} is the Ricci scalar associated to the metric \tilde{g}_{AB} .

Thus, the action (55) becomes

$$\begin{aligned} S = & -\frac{1}{16\pi G_{p+1}} \int d^{p+1} y \sqrt{-\tilde{g}} \Pi a_s e^{-\phi} \{ \varphi (-2a^i D^A \partial_A a_i - a^i a^j \partial_A a_i \partial^A a_j \\ & + a^i a^j \partial^A a_k \partial_A a_l \eta_{ij}^{kl} + \partial^A \phi \partial_A \phi + \tilde{R}) + 2\Lambda \varphi^a \}, \end{aligned} \quad (63)$$

where G_{p+1} is the Newton constant in $p + 1$ dimensions. The relation between G_{p+1} and G_{D+1} is

$$\frac{1}{G_{p+1}} = \frac{V_d}{G_{D+1}}, \quad (64)$$

where V_d is a volume element in $d = D - p$ dimensions. This action can be rewritten as

$$\begin{aligned} S = & -\frac{1}{16\pi G_{p+1}} \int d^{p+1} y \sqrt{-\tilde{g}} D^A J_A \\ & -\frac{1}{16\pi G_{p+1}} \int d^{p+1} y \sqrt{-\tilde{g}} \Pi a_s e^{-\phi} \{ \varphi (a^i a^j \partial^A a_i \partial_A a_j - 2\partial^A \phi a^i \partial_A a_i \\ & + \partial^A \phi \partial_A \phi - a^i a^j \partial^A a_k \partial_A a_l \eta_{ij}^{kl}) + 2\Lambda \varphi^a \} \\ & -\frac{1}{16\pi G_{p+1}} \int d^{p+1} y \sqrt{-\tilde{g}} \Pi a_s e^{-\phi} \varphi \tilde{R} + \frac{1}{16\pi G_{p+1}} \int d^{p+1} y \sqrt{-\tilde{g}} J_A \varphi^{-1} \partial^A \varphi, \end{aligned} \quad (65)$$

where

$$J_A = (-2\Pi a_s e^{-\phi} \varphi a^i \partial_A a_i). \quad (66)$$

Dropping the total derivative, the action (65) is reduced to

$$\begin{aligned} S = & -\frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} \Pi a_s e^{-\phi} \{ \varphi (a^i a^j \partial^A a_i \partial_A a_j - 2\partial^A \phi a^i \partial_A a_i + \\ & + \partial^A \phi \partial_A \phi - a^i a^j \partial^A a_k \partial_A a_l \eta_{ij}^{kl}) + 2\Lambda \varphi^a \} \end{aligned} \quad (67)$$

$$-\frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} \Pi a_s e^{-\phi} \varphi \tilde{R} + \frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} J_A \varphi^{-1} \partial^A \varphi.$$

If we define the coordinate x^0 as

$$\Pi a_s e^{-\phi} = e^{-x^0}, \quad (68)$$

the brane coupling “constant” T_p in the form

$$\frac{e^{-x^0}}{16\pi G_{p+1}} = \frac{1}{2T_p} \quad (69)$$

and the variables x^i as

$$x^i \equiv \ln a_i \quad (70)$$

we find that (67) becomes

$$\begin{aligned} S = & \frac{1}{2} \int \frac{d^{p+1}y}{T_p} \sqrt{-\tilde{g}} \varphi (\tilde{g}^{AB} \partial_A x^{\hat{\mu}} \partial_B x^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}}) - 2\Lambda \varphi^a \\ & - \frac{1}{2} \int \frac{d^{p+1}y}{T_p} \sqrt{-\tilde{g}} \varphi \tilde{R} + \frac{1}{16\pi G_{p+1}} \int d^{p+1}y \sqrt{-\tilde{g}} J_A \varphi^{-1} \partial^A \varphi. \end{aligned} \quad (71)$$

where $\eta_{\hat{\mu}\hat{\nu}} = \text{diag}(-1, 1, \dots, 1)$. Here the indices $\hat{\mu}, \hat{\nu}, \dots$ etc run from 0 to $d = D - p$.

Let us assume now the case in which we can drop the last two terms from (71). Setting the constant Λ as

$$\Lambda = \frac{p-1}{2} \quad (72)$$

and the quantity a as

$$a = \frac{p+1}{p-1} \quad (73)$$

we obtain

$$S = \frac{1}{2} \int \frac{d^{p+1}y}{T_p} \sqrt{-\tilde{g}} (\varphi \tilde{g}^{AB} h_{AB} - (p-1) \varphi^{\frac{p+1}{p-1}}), \quad (74)$$

where

$$h_{AB} = \partial_A x^{\hat{\mu}} \partial_B x^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}}. \quad (75)$$

We recognize (74) as the action $S_p^{W_3}$ given in (21). Integrating out φ we find

$$\varphi = \left(\frac{\tilde{g}^{AB} h_{AB}}{p+1} \right)^{\frac{p-1}{2}}. \quad (76)$$

Substituting this result for φ into (75) we get

$$S = \frac{1}{(p+1)^{\frac{p+1}{2}}} \int \frac{d^{p+1}y}{T_p} \sqrt{-\tilde{g}} (\tilde{g}^{AB} h_{AB})^{\frac{p+1}{2}} \quad (77)$$

which is the Weyl invariant action for p-branes. Therefore, we have shown that the Weyl invariant action (77) follows from the gravity-dilaton action (55) when the constants Λ and a have the form (72) and (73) respectively, that is, we have shown a mechanism to derive (74) from the following action

$$S = -\frac{1}{16\pi G_{D+1}} \int d^{D+1}y \sqrt{-g} e^{-\phi} (\varphi (R + (\nabla\phi)^2) + (p-1) \varphi^{\frac{p+1}{p-1}}), \quad (78)$$

Integrating out φ in (78) we derive the action

$$S = -\frac{1}{16\pi G_{D+1}(p+1)^{\frac{p+1}{2}}} \int d^{D+1}y \sqrt{-g} e^{-\phi} (R + (\nabla\phi)^2)^{\frac{p+1}{2}}, \quad (81)$$

Therefore, we have a higher curvature theory as an starting point. It is interesting to note that only for the case $p = 1$, corresponding to strings, (81) turns out to be an action linear in curvature tensor R .

6.- FINAL COMMENTS

In this article we have clarified some aspects of the Weyl invariant actions for p-branes, Dp-branes and null p-branes. In sections 2, 3 and 4 we develop a mechanism to derive different Weyl invariant p-branes and Dp-branes actions. While in section 5 we have shown a geometric origin of Weyl invariant actions for p-branes.

So far, our treatment of the equivalence of various Weyl invariant actions has been classical, without taking into account certain determinants of the associated partition function. Such determinants may lead to some anomalies allowing equivalence only under certain conditions. For instance, at the quantum level the Polyakov type action for p-branes (7) is equivalent to the Dirac-Nambu-Goto type action (1) only if we set $d = 25$.

The supersymmetrization of the procedure proposed in second, third and fourth sections seem to be straightforward. One needs simply to use the prescription

$$\partial_A x^{\hat{\mu}} \rightarrow E_A^{\hat{\mu}} = \partial_A Z^{\hat{M}} \Pi_{\hat{M}}^{\hat{\mu}}, \quad (82)$$

where $Z^{\hat{M}} = (x^{\hat{\mu}}, \theta^{\hat{\alpha}})$ are the coordinates of a superspace and $\Pi_{\hat{M}}^{\hat{\mu}}$ are the supervielbein. In this case the action (13) can be generalized to

$$S_p^{W_1} = -\frac{T_p}{2} \int d^{p+1} \xi \sqrt{-g} (\Phi^{\frac{p-1}{2}} g^{AB} E_A^{\hat{\mu}} E_B^{\hat{\nu}} \eta_{\hat{\mu}\hat{\nu}} - (p-1) \Phi^{\frac{p+1}{2}}) \quad (83)$$

$$+ \frac{1}{(p+1)!} \varepsilon^{A_1 \dots A_{p+1}} E_{A_1}^{\hat{\mu}_1} \dots E_{A_{p+1}}^{\hat{\mu}_{p+1}} A_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}},$$

where $A_{\hat{\mu}_1 \dots \hat{\mu}_{p+1}}$ is a completely antisymmetric gauge field tensor. This action is invariant under the world volume diffeomorphisms and the Weyl transformation (14)-(15).

It may be interesting for further works to follow similar procedure as the one of section 5 to find the super p-branes actions from a higher dimensional supergravity action. For instance, this may help to understand the so called κ symmetry from a geometrical perspective.

Recently, a Weyl invariant spinning membrane action [18] was constructed where the conformal symmetry and S-supersymmetry are broken. It may be interesting to relate the present work with such a system.

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REFERENCES

- [1] J. Polchinski, *String Theory*, VOL. I and II, (Cambridge, UK: Univ. Press (1998)).
- [2] J. A. Nieto Classical Test Particles and (4+N)-Dimensional Theories of Space Time, PhD Thesis, University of Texas at Austin (1986); J. A. Nieto, C. Nuñez and L. C. Shepley, “Dynamical Cosmological Constant from a Four-Index Antisymmetric Gauge Field”, unpublished (1986) Austin Texas.
- [3] M. S. Alves and J. Barcelos Neto, *Europhys. Lett* **7**, 395 (1988), (Erratum-ibid. 8:90,1989) ; U. Lindstrom and G. Theodoridis *Phys. Lett.* **B208**, 407 (1988); J. A. Nieto, *Rev. Mex. Fis.* **34**, 597 (1988).
- [4] U. Lindstrom, M. Rocek, *Phys. Lett.* **B218**, 207 (1989); A. Karlhede and U. Lindstrom, *Phys. Lett.* **B209**, 441 (1988); C. Alvear, R. Amorim and J. Barcelos-Neto, *Phys. Lett* **B273**, 415 (1991); J. Barcelos-Neto, *Phys. Lett.* **B245**, 26 (1990), Addendum-ibid. **B249**, 551 (1990); J. A. Nieto and C. Nuñez , *Nuovo Cim.* **B106**, 1045 (1991); L. Cendejas and J. A. Nieto, *Nuovo Cim.* **B105**, 1159 (1990); E.R.C. Abraham, P.K. Townsend and P.S. Howe, *Class. Quant. Grav.* **6**, 1541 (1989).
- [5] M. Cavaglia, *Mod. Phys. Lett.* **A15**, 2113 (2000); hep-th/0011136.
- [6] J. A. García, R. Linares, J. D. Vergara, *Phys. Lett.* **B503**, 154 (2001); hep-th/0011085
- [7] J. A. Nieto and C. M. Yee, *Mod. Phys. Lett.* **A15**, 1611 (2000).
- [8] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [9] H. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phys. Lett.* **B429**, 263 (1998); *Phys. Rev.* **D59** 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phys. Lett.* **B436**, 257 (1998); N. Arkani-Hamed, S. Dimopoulos, and J. March-Russell, hep-th/9809124.
- [10] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory Vol. I and II* (Cambridge,

Uk: Univ. Press (1987))

- [11] E. Witten, Nucl. Phys. **B443**, 85 (1995); P. Horava and E. Witten, ibid. **B460**, 506 (1995); **B475**, 94 (1996).
- [12] U. Lindstrom, Int. J. Mod. Phys. **A3**, 2401 (1988).
- [13] B. Brinne, S. E. Hjelmeland and Ulf Lindstrom Phys. Lett. **B459**, 507 (1999); hep-th/9904175.
- [14] M. Abou Zeid and C. M. Hull, Phys. Lett. **B404**, 264 (1997); hep-th/9802179.
- [15] Schild, Phys. Rev. **D16**, 1560 (1977).
- [16] J. A. Nieto, Rev. Mex. Fis. **36**, S204 (1990).
- [17] M. Gasperini; *Elementary Introduction to Pre-Big-Bang Cosmology and to the Relic Graviton Background*, proceedings of SIGRAV Graduate School in Contemporary Relativity and Gravitational Physics, Villa Olmo, Como, Italy, 19-24 Apr 1999; hep-th/9907067
- [18] C. Castro, *Remarks on the Existence of Spinning Membrane Action*: hep-th/0007031